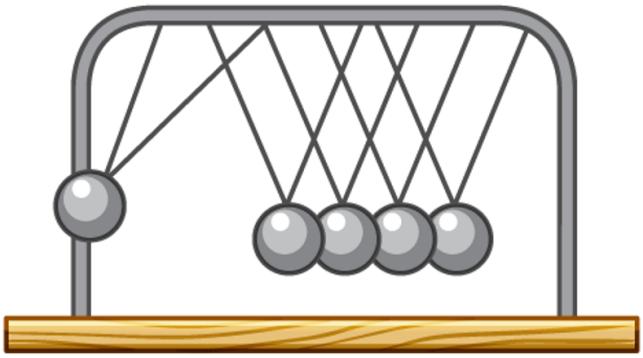
Chapter 01: Motion in one dimension and plane

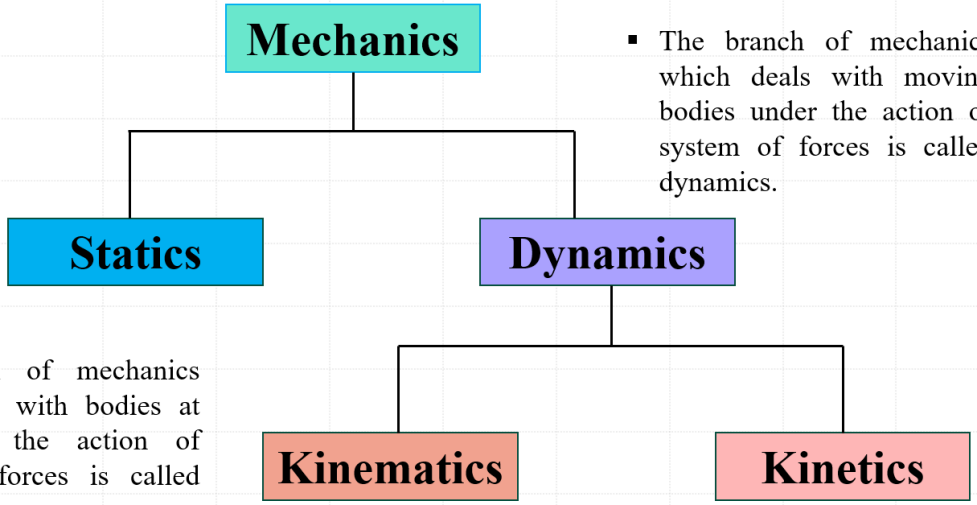
Particle kinematics

Mechanics

- Mechanics is the branch of Physics dealing with the study of motion when subjected to forces or displacements, and the subsequent effects of the bodies on their environment.



Types of Mechanics



- The branch of mechanics which deals with moving bodies under the action of system of forces is called dynamics.
- The branch of mechanics which deals with bodies at rest under the action of system of forces is called statics.

Kinematics: Kinematics is the branch of mechanics that deals with motion without considering the forces that cause it.

- It is an essential aspect of engineering and physics, used to describe, predict and understand motions. This includes the motion of objects as well as the motion of particles.

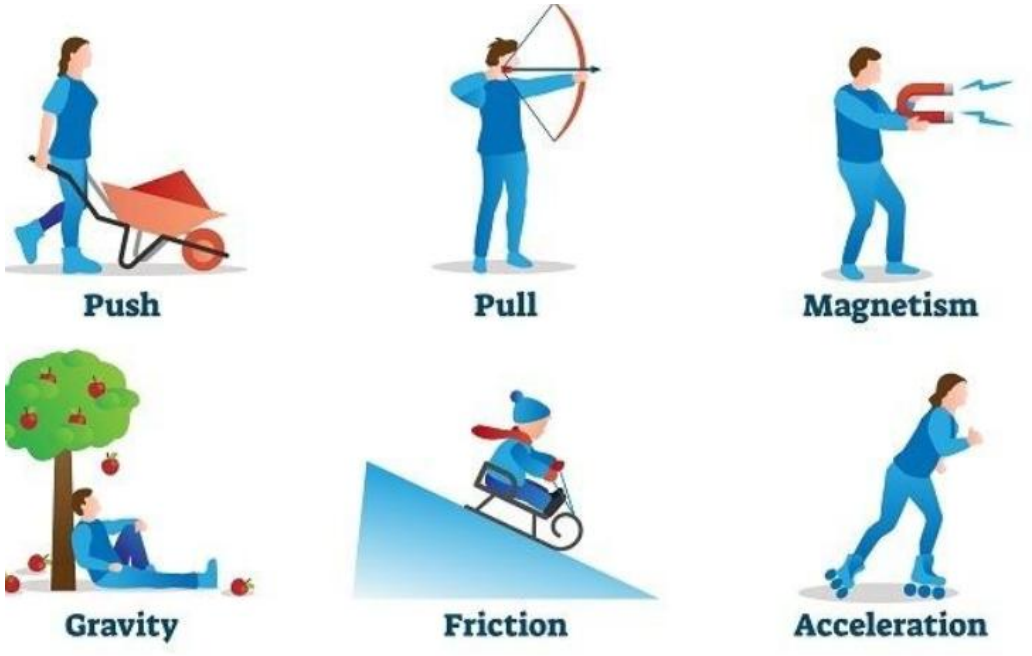
Examples of Kinematics in Real-Life Situations

There are countless examples of kinematics in the world around us. Here are just a few:

1. A baseball is being thrown
2. A person riding a bike
3. A stone being skipped on water
4. A child swinging on a playground swing
5. A pendulum swinging back and forth
6. Someone is doing a somersault

Kinetics: A branch of Mechanics that deals with the effects of forces upon the motions of material bodies or with changes in a physical system.

- For instance, when you throw a ball in the air, kinetics explain the friction that causes the throw. Kinematics explains the acceleration, speed and final position of the ball when it falls in the ground.



Difference between Kinetics and Kinematics	
Kinetics	**Kinematics**
1.Studies **the cause for the motions** of the object	1. Studies the **position, acceleration and speed** of an object
2. The **mass of the object** is taken into consideration	2. The **mass of the object is not taken into consideration**
3. Its application lies in the design of automobiles	3. It is applied to study the movement of celestial bodies
4. **Force** is considered	4. Force is not considered
5. It does not involve any mathematical expressions	5. It involves **mathematical expressions**

Motion

- When a body or an object changes its position in course of time with respect to its surroundings is said to be in motion and that object is called moving object.

There may be five different kinds of motion:

1. Translational motion (example: a falling stone from a certain height)
2. Rotational motion (example: motion of an electric fan, motion of hour or minute hand of a clock)
3. Transla-rotational motion (example: motion of the wheel of a cart, motion of the wheel of a bicycle)
4. Periodic motion (example: the motion of the earth round the sun, piston of a cylinder in a car) and
5. Vibrational motion (example: pendulum of wall-clock)

Equations of Motion

- $v = u+at$
- $s = \{(u+v)/2\}t$
- $v^2 = u^2+2as$
- $s = ut+1/2at^2$
- $s = vt$ (for uniform velocity)

Freely Falling Bodies:

A **freely falling body** is an object that is falling under the influence of **gravity alone**, without any air resistance or any other force acting on it. Near the surface of the Earth, the acceleration due to gravity **g** is approximately **9.8 m/s²** (downward).

In ideal conditions (ignoring air resistance):

- All objects, regardless of their mass, fall with the **same acceleration** due to gravity.
- The motion is uniformly accelerated motion with acceleration **g** directed downward.

Equations of Motion for Freely Falling Bodies:

These equations are derived from the general equations of motion under constant acceleration, replacing the acceleration **a** with **g**.

When falling downward (taking downward as positive direction):

- $v = u + gt$
- $v^2 = u^2 + 2gs$
- $s = ut + 1/2gt^2$

Upwardly thrown object

1. If a piece of stone is thrown vertically upward with velocity 196 ms^{-1} , (a) how long does it rise? (b) what will be its velocity after 5 s? (c) how long it will take to come back to its starting point? (Acceleration due to gravity, 9.8 ms^{-2}).

(a) How long does it rise?

Answer:

Let the required time t

We know that, $t = \frac{u}{g}$

$$\Longrightarrow t = \frac{196}{9.8}$$

$$\Longrightarrow t = 20\ s$$

(b) What will be its velocity after 5 s?

Answer:

We know that, $v = u - gt$

$\Longrightarrow v = 196 - (9.8 \times 5)$

$\Longrightarrow v = 147$ ms^{-1}

(c) How long it will take to come back to its starting point?

Answer:

Let the required time T

Therefore, $T = \frac{2u}{g}$

$\Longrightarrow T = \frac{2 \times 196}{9.8}$

$\Longrightarrow T = 2 \times 20$

$\Longrightarrow T = 40$ s

Freely Falling Object

2. A body is thrown from the top of a tower of 200 m height. At the same time another object is thrown vertically upward with a velocity of 50 ms^{-1}. When and where will the bodies meet?

Answer:

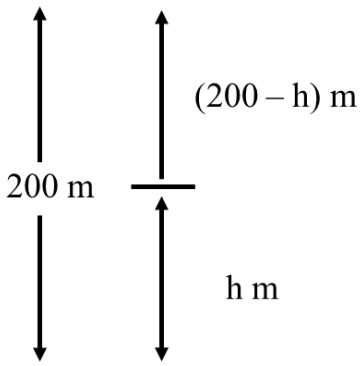
Let after time t the two bodies meet at a height h from the ground.

Therefore, for the first body we get,

height $= ut + \frac{1}{2} gt^2$

$\Longrightarrow (200 - h) = 0 + \frac{1}{2} gt^2$

$\Longrightarrow (200 - h) = \frac{1}{2} gt^2$ …….. (**1**)



For the second body,

$$\text{height } = ut - \frac{1}{2} gt^2$$

$$\Longrightarrow h = ut - \frac{1}{2} gt^2 \text{ ..... (2)}$$

Now adding equations (1) and (2) we get,

$ut = 200$

$\Longrightarrow 50 \times t = 200$

$\Longrightarrow 50 \times t = 200$

$\Longrightarrow t = \frac{200}{50}$

$\Longrightarrow t = 4$ s

Here,

$u = 50$ ms^{-1}

Inserting this value of t in equation (1) we get

$$(200 - h) = \frac{1}{2} 9.8 \times 4^2$$

$$\Longrightarrow 200 - h = 78.4$$

$$\Longrightarrow h = 121.6\, m$$

Projectile Motion

Projectile motion is the motion of an object that is thrown or projected into the air, subject only to the acceleration due to gravity. The path followed by the projectile is called a **trajectory** and it is typically a **parabola** (if air resistance is neglected).

Examples:

- A football kicked at an angle
- A cannonball fired from a cannon
- A stone thrown into the air at an angle

➢ Verify that the trajectory of a projectile is parabolic in nature

Projectile motion is described in terms of motion in the horizontal or x-direction and independently by motion in the vertical or y-direction. If air resistance is ignored the horizontal motion continues at constant velocity because its horizontal acceleration is zero. The vertical motion will have acceleration due to gravity.

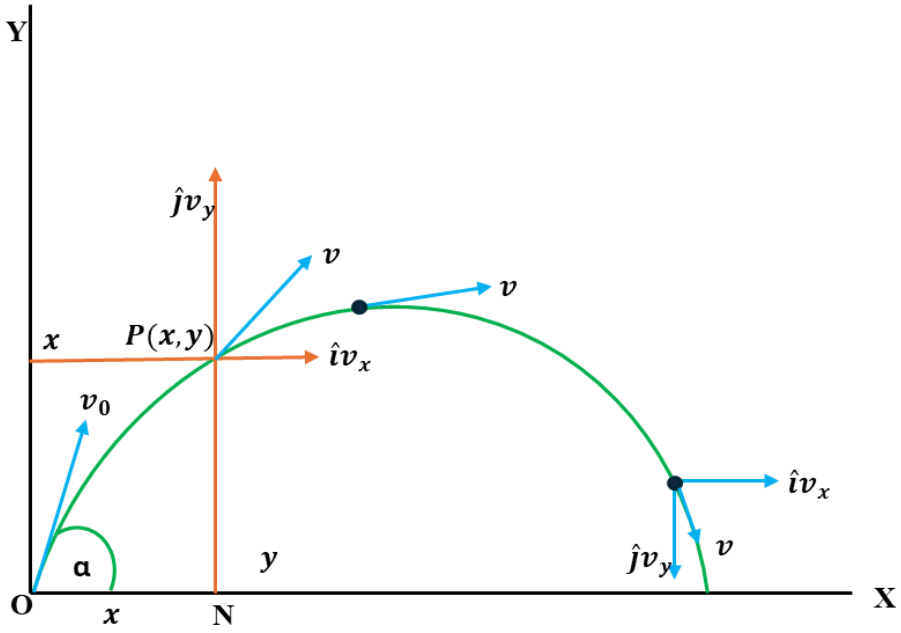


Fig. 7

Let a projectile begin its flight from a point O with initial velocity v_0 and making an angle α with the horizontal direction **(Fig. 7).** Taking O as origin let the horizontal and vertical directions be considered along X and Y-axes. So, at t = 0, the horizontal component of initial velocity,

$v_{x_0} = v_0 cos\alpha$ and vertical component, $v_{y_0} = v_0 sin\alpha$

Now, from the equation of motion,

$$v_x = v_{x_0} + a_x t$$

Since, $a_x = 0$, we get, $v_x = v_{x_0} = v_0 cos\alpha$

Let at t = t, the projectile reaches the point P, whose co-ordinate is (x, y) and where its velocity is $\vec{v}$. So, the displacement of the projectile parallel to the ground i.e., along X-axis is,

$$x = ON = v_0 cos\alpha \times t$$

$$\text{or, } t = \frac{x}{v_0 cos\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now, the vertical component of the velocity is,

$$v_y = v_{y_0} + a_y t$$

Since, a_y is $+ve$ in the upward, but the direction of g is downward, so g is negative.

$\therefore v_y = v_0 sin\alpha - gt \qquad [\because a_y = -g]$

So, the vertical displacement at t = t,

$$y = PN = y_0 + v_{y_0} t + \frac{1}{2} a_y t^2$$

or, $y = 0 + v_0 sin\alpha \times t - \frac{1}{2} g t^2$

$\therefore y = v_0 sin\alpha t - \frac{1}{2} g t^2$ ..... (2)

Putting the value of t from equation (1) into equation (2) we get,

$y = v_0 sin\alpha \times \frac{x}{v_0 cos\alpha} - \frac{1}{2} g \left(\frac{x}{v_0 cos\alpha}\right)^2$

or, $y = x tan\alpha - \frac{1}{2} g \frac{x^2}{v_0^2 cos^2\alpha}$

$\therefore y = x tan\alpha - \frac{g x^2}{2 v_0^2 cos^2\alpha}$ ..... (3)

In equation (3) $\alpha, g,$ and v_0 are constants. So, taking $tan\alpha = b$ and $\frac{g}{2 v_0^2 cos^2\alpha} = c$ as constants equation (3) can be written as,

$\boldsymbol{y = bx - cx^2}$ ..... (4)

This is an equation of a parabola. Hence the **path of motion of a projectile is parabolic**.

Now, we will derive some important expressions relating to projectile motion.

(i) Maximum height of the path of a projectile

We know from equation of motion,

$$v^2 = v_0^2 + 2as$$

Since the vertical component of the projectile is along Y-axes, so the above equation reduces to,

$v_y^2 = v_{y_0}^2 + 2 a_y y$ ..... (a)

Let the maximum height reached by the projectile be H where final velocity = 0 i.e., $v_y = 0$. So, we get from equation (a),

$0 = v_{y_0}^2 - 2gH \qquad [\because a_y = -g]$

But $v_{y_0} = v_0 sin\alpha$

$$\therefore 0 = v_0^2 sin^2\alpha - 2gH$$

or, $H = \frac{v_0^2 sin^2\alpha}{2g}$ ..... (5)

when $\alpha = 90°$, the height will be maximum. So,

$\boldsymbol{H = \frac{v_0^2}{2g}}$ ..... **(6)**

Knowing v_0 *and* g, H can be determined.

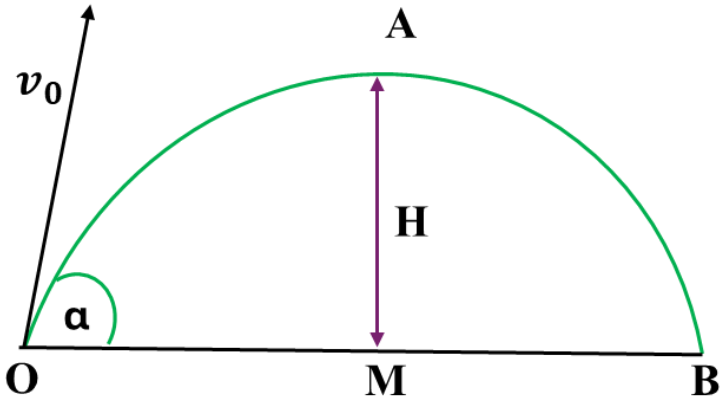


Fig. 8

(ii) Time to Reach Maximum Height

From equation of motion we know,

$$v = v_0 + at$$

Along Y-axes i.e., along vertical direction,

$v_y = v_{y_0} + a_y t$ ..... (b)

Now the vertical component of initial velocity $v_{y_0} = v_0 \sin\alpha$ and at maximum height final velocity $v = 0$. So, from equation (b) we get,

$0 = v_0 sin\alpha - gt$ $\quad [\because a_y = -g]$

or, $\boldsymbol{t = \frac{v_0 sin\alpha}{g}}$ ..... **(7)**

Hence, by knowing v_0 , α *and* g we can find the time t.

(iii) Time of Flight

Let the time of flight be T. Now, we know the time of ascend to the maximum height = time of descend to the ground. So,

$$T = t + t = 2t$$

Using equation (7), we get,

$$\boldsymbol{T = \frac{2v_0 sin\alpha}{g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(8)}$$

(iv) Horizontal Range

The linear distance from the point of projection to the end of flight is called the horizontal range. Alternatively, the distance travelled along the horizontal direction in the time of flight is called the horizontal range. This is represented by R.

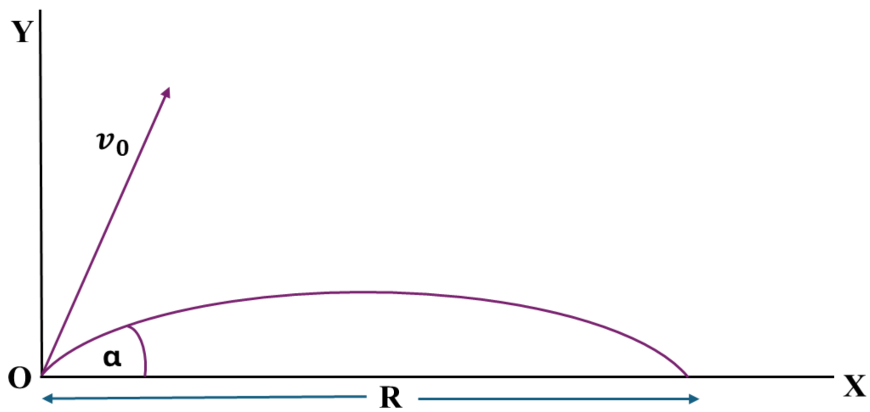


Fig. 9

∴ R = horizontal component of the initial velocity × time of flight

$$or,\ R = v_0 cos\alpha \times T$$

$$or,\ R = v_0 cos\alpha \times \frac{2v_0 sin\alpha}{g}$$

$$or, R = \frac{v_0^2 \ .\ 2sin\alpha.\ cos\alpha}{g}$$

$$\therefore \boldsymbol{R = \frac{v_0^2\ sin2\alpha}{g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(9)}$$

By knowing v_0 , α and g we can find the time R.

R will be maximum when $sin2\alpha = 1$

$$or, sin2\alpha = sin90°$$

$$or, 2\alpha = 90°$$

$$\therefore \alpha = 45°$$

In that case, $\boldsymbol{R_{max} = \frac{v_0^2}{g}}$ ..... (10)

If an object is thrown at an angle of 45° with the horizontal direction, the horizontal range will be maximum.

3. A football is kicked making an angle 45° with the ground at velocity $60ms^{-1}$.

(a) Find the magnitude of velocity of the football after 5sec.

(b) find the maximum height and horizontal range of the football.

Answer:

(a) Let the point from where the ball is kicked is the origin and Y-axis is positive vertically upward.

Now, let v_x and v_y are the horizontal and vertical components of $\vec{v}$.

$\therefore v = \sqrt{v_x^2 + v_y^2}$

Here,

Angle of projection, $\theta = 45°$

Initial velocity, $v_0 = 60ms^{-1}$

Time, t= $5sec$

Final velocity, v =?

Now, horizontal and vertical components of initial velocity v_0 are v_{x0} and v_{y0} respectively.

We get,

$$v_x = v_{x0} + a_x t$$

$$\Rightarrow v_x = v_0 \cos\theta + a_x t$$

$$\Rightarrow v_x = v_0 \cos\theta \qquad [\text{ since, } a_x = 0\ ]$$

$$\Rightarrow v_x = 60 \cos 45°$$

$$\Rightarrow v_x = 42.43\ ms^{-1}$$

$$And, v_y = v_{y0} + a_y t$$

$$\Rightarrow v_y = v_0 \sin\theta + a_y t$$

$$\Longrightarrow v_y = 60\ sin45° + (-9.8) \times 5 \qquad [\text{since, } a_y = -g = -9.8\ ]$$

$$\Longrightarrow v_y = 6.57\ ms^{-1}$$

$\therefore v = \sqrt{v_x^2 + v_y^2}$

$\Longrightarrow v = \sqrt{(42.43)^2 + (6.57)^2}$

$\Longrightarrow v = \sqrt{1800.3049 + 43.1649}$

$\Longrightarrow v = 42.94\ ms^{-1}$

(b) We know, maximum height, $H = \frac{v_0^2 sin^2\theta}{2g}$

$$\Longrightarrow H = \frac{(60)^2 sin^2 45°}{2 \times 9.8}$$

$$\Longrightarrow H = \frac{1800}{19.6}$$

$$\Longrightarrow H = 91.84\ m$$

Horizontal range, $R = \frac{v_0^2\ sin2\theta}{g}$

$$\Longrightarrow R = \frac{(60)^2 \times \sin(2 \times 45°)}{9.8}$$

$$\Longrightarrow R = 367.347\ m$$

4. A ball is projected at an angle of 30° with the horizontal with a velocity of $40ms^{-1}$ from the top of a tower of height 50 m.

(a) when and what distance from the bottom of the tower will the ball touch the ground?

(b) also calculate the velocity of the ball when it reaches the ground. ($g = 10\ ms^{-2}$)

Answer:

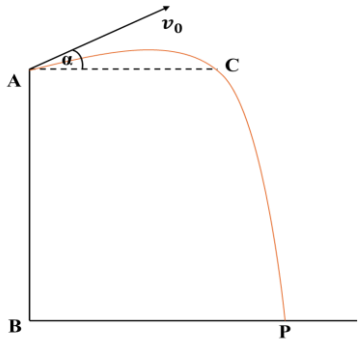
(a) Here,
Angle of projection, $\theta = 30°$

Initial velocity, $v_0 = 40ms^{-1}$

Height of the tower, H= $50m$

Time, t=?

Distance, x =?



We know horizontal and vertical components of the initial velocities are,

$$v_{x0} = v_0 \cos\theta$$

$$\Rightarrow v_{x0} = 40 \cos 30°$$

$$\Rightarrow v_{x0} = 34.64\, ms^{-1}$$

And $v_{y0} = v_0 \sin\theta$

$$\Rightarrow v_{y0} = 40 \sin 30°$$

$$\Rightarrow v_{y0} = 20\, ms^{-1}$$

If the ball strikes the ground after t sec., then,

$$h = v_{y0}\, t + \frac{1}{2} g t^2$$

$\Rightarrow h = -20t + \frac{1}{2} \times 10 \times t^2$ [since, v_{y0} *is downward, hence* $-ve$]

$\Rightarrow 5t^2 - 20t - 50 = 0$

$\Rightarrow 5(t^2 - 4t - 10) = 0$

$\Rightarrow (t^2 - 4t - 10) = 0$

$\Rightarrow t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4.1.(-10)}}{2 \times 1}$

$\Rightarrow t = \frac{11.48}{2}$ [taking the positive sign only]

$\therefore t = 5.7\, sec$

So the ball reaches the ground after 5.7 sec.

Horizontal distance from the bottom of the tower = BP

$$\therefore BP = x = v_{x0}t$$

$$\Longrightarrow x = 34.64 \times 5.7$$

$$\Longrightarrow x = 197.45\, m$$

(b) The vertical velocity of the ball at the time of striking the ground is,

$$v_y = v_{y0} + gt$$

$$\Longrightarrow v_y = -20 + 10 \times 5.7$$

$$\Longrightarrow v_y = 37\, ms^{-1}$$

Horizontal velocity,

$$v_x = v_{x0}$$

$$= 34.64\, ms^{-1}$$

$\therefore v = \sqrt{v_x^2 + v_y^2}$

$\Longrightarrow v = \sqrt{(34.64)^2 + (37)^2}$

$\Longrightarrow v = 50.68 ms^{-1}$

Relative Velocity

The velocity of an object **with respect to another moving object** is called **relative velocity**. It tells us **how fast one object appears to be moving from the viewpoint of another**.

Formula (in linear motion):

If object **A** has velocity $\vec{v}_A$ and object **B** has velocity $\vec{v}_B$ (both measured relative to the ground), then the **velocity of A relative to B** is:

$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

Similarly, the **velocity of B relative to A** is:

$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$

Relative Acceleration

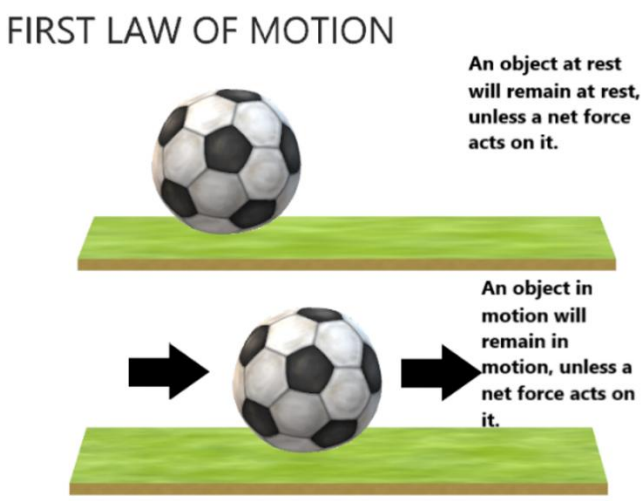
The acceleration of one object with respect to another is called **relative acceleration**.

Chapter -02: Particles Dynamics

NEWTON'S FIRST LAW OF MOTION: INERTIA

- **Every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force.**

Example: A ball rolling down a hill will continue to roll unless friction or another force stops it.

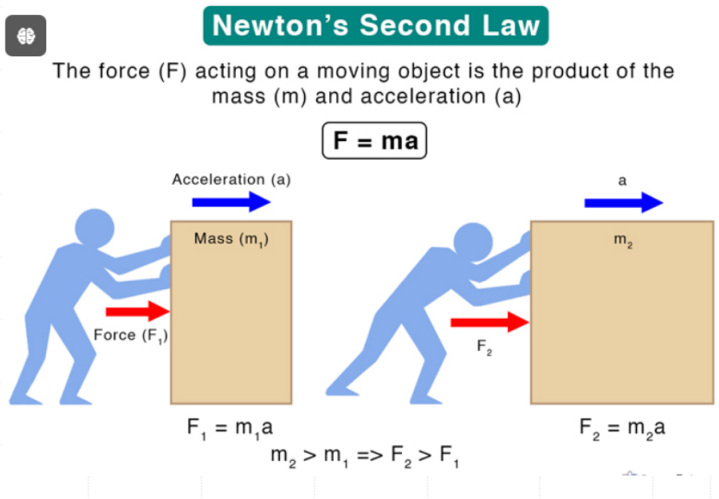


NEWTON'S SECOND LAW OF MOTION: FORCE

- **The acceleration of an object depends on the mass of the object and the amount of force applied.**

Example of force involving aerodynamics:

•An aircraft's motion resulting from aerodynamic forces, aircraft weight, and thrust.



Newton's Second Law of Motion (Statement):

The rate of change of momentum of a body is directly proportional to the net external force acting on it and takes place in the direction of that force.

Mathematically,

$F \propto dp/dt$

or, $F = dp/dt$

For a body of constant mass, $F = ma$

where:

- (F) = net external force,
- (m) = mass of the body,
- (a) = acceleration produced.

Derivation of Newton's First Law from Newton's Second Law

Newton's Second Law states that:

$F = ma$

or, more generally,

$F = dp/dt$

where F is the net external force acting on a body.

Using the force–acceleration form:

$F=ma$

If the resultant external force on a body is zero, $(F=0)$

Therefore, $ma = 0$

For a body having finite mass $(m \neq 0)$,

$a = 0$

Since acceleration is the rate of change of velocity,

$a = dv/dt = 0$

Thus, $dv = 0$

which implies that velocity remains constant.

A constant velocity means:

- If the initial velocity is zero, the body remains at rest.
- If the initial velocity is non-zero, the body continues to move with uniform velocity in a straight line.

Hence, we obtain **Newton's First Law of Motion**:

A body remains at rest or continues to move with uniform velocity in a straight line unless acted upon by a net external force.

Newton's Third Law:
Action & Reaction

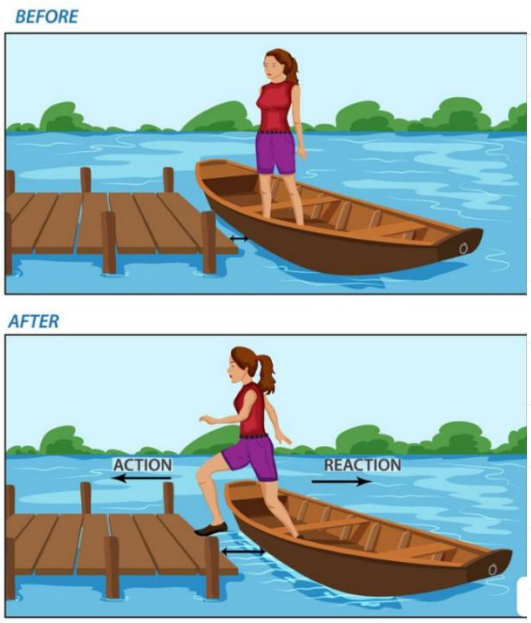
- **Whenever one object exerts a force on a second object, the second object exerts an equal and opposite force on the first.**

Examples of action and reaction involving aerodynamics:

•The motion of lift from an airfoil, the air is deflected downward by the airfoil's action, and in reaction, the wing is pushed upward.

•The motion of a spinning ball, the air is deflected to one side, and the ball reacts by moving in the opposite direction.

•The motion of a **jet engine** produces thrust and hot exhaust gases flow out the back of the engine, and a thrusting force is produced in the opposite direction



Mathematical representation of 3 laws of motion:

First Law	**$F_{net} = 0$ if velocity is constant**
Second Law	**F=ma**
Third Law	**$F_{AB}=-F_{BA}$**

Applications of Newton's First Law of Motion In Our Daily Life:







Newton's laws of motion have several limitations:

1.Applicability: They are valid only in **inertial frames of reference**, meaning they do not account for frames that are accelerating. In such cases, the laws need modifications or different approaches.

2.Non-relativistic: They do not hold true at very high speeds (close to the speed of light) or in strong gravitational fields, where relativistic effects become significant.

3.Quantum effects: Newton's laws fail to adequately describe the behavior of particles at atomic and subatomic scales, where quantum mechanics takes precedence.

Frictional forces

Frictional Forces refer to the resistive forces that oppose the relative motion or the tendency of such motion between two surfaces in contact. It plays a crucial role in everyday life—without friction, we couldn't walk, drive, or even hold objects.

Types of Frictional Forces

1. **Static Friction (f_s):**
   - Acts when two surfaces are in contact but not moving relative to each other.
   - Prevents motion.
   - Has a maximum value:
     $f_s^{max} = \mu_s \mathrm{N}$

where:

μ_s = coefficient of static friction
N = normal force

2. **Kinetic (or Dynamic) Friction (f_k):**
   - Acts when two surfaces are sliding past each other.
   - Generally less than static friction.
   - Given by:
     $f_k = \mu_k \mathrm{N}$

where:

μ_k = coefficient of kinetic friction

1. **Rolling Friction**:
   - Resistance when an object **rolls** over a surface (e.g., a ball or wheel).
   - Much smaller than kinetic friction.
2. **Fluid Friction** (or **drag**):
   - Resistance when an object moves through a fluid (liquid or gas).

Factors Affecting Friction

- **Nature of surfaces**: Rougher surfaces produce more friction.
- **Normal force**: Greater force pressing the surfaces together increases friction.
- **Lubrication**: Reduces friction.
- **Surface area** (in rigid body cases, surprisingly, does not directly affect friction).

Importance of Friction

- Allow walking and running.
- Enables cars to move and stop.
- Provides grip for tools.
- Generates heat in machines (sometimes undesirable).

Dynamics of Uniform Circular Motion

Uniform Circular Motion (UCM) refers to the motion of an object moving at a constant speed along a circular path. Although the speed is constant, the velocity is not, because the direction of motion is continuously changing — and so is the acceleration.

1. Velocity in UCM

- **Magnitude (speed)** is constant.
- **Direction** is always tangent to the circular path.
- Velocity vector keeps changing → implies **acceleration is present**.

2. Centripetal Acceleration (a_c)

- Directed **toward the center** of the circular path.
- Caused by continuous change in direction of velocity.
- Magnitude:

$$a_c = \frac{v^2}{r}$$

where:

v = speed of the object

r = radius of the circle

3. Centripetal Force (F_c)

- **Net force** required to keep an object in circular motion.
- Also directed **toward the center** of the circle.
- Given by:

$$\boldsymbol{F_c = ma_c = m\frac{v^2}{r}}$$

where:

m = mass of the object, v = speed, r = radius

This force is not a new kind of force—it's provided by existing forces (tension, gravity, friction, etc.), depending on the system.

- **Examples of Forces Acting as Centripetal Force**

System	Centripetal Force Provided By
Car turning in a circle	Friction between tires and road
Ball on a string (circular path)	Tension in the string
Planet orbiting the Sun	Gravitational force
Electron around nucleus	Electrostatic attraction

- **Can an object have a constant velocity while undergoing circular acceleration? Provide an example to illustrate your explanation.**

No, an object cannot have a constant velocity while undergoing circular acceleration. Although the object may have a constant speed, its velocity is not constant due to the continuous change in direction. For example, a car moving at a constant speed around a circular track is constantly changing direction, resulting in centripetal acceleration. This means the velocity is changing even though the speed remains the same.

Chapter -03: Work and Energy

Work done by constant force

3. Work Energy Theorem

The principle of work and kinetic energy (also known as the work-energy theorem) states that the work done by the sum of all forces acting on a particle equals the change in the kinetic energy of the particle.

Derivation

For the sake of simplicity, we will consider the case in which the resultant force F is constant in both magnitude and direction and is parallel to the velocity of the particle. The particle is moving with constant acceleration $\boldsymbol{a}$ along a straight line. The relationship between the net force and the acceleration is given by the equation $F = ma$ (Newton's second law), and the particle's displacement d, can be determined from the equation:

$$v_f^2 = v_i^2 + 2ad \tag{6}$$

obtaining,

$$d = \frac{v_f^2 - v_i^2}{2a} \tag{7}$$

The work of the net force is calculated as the product of its magnitude (F=ma) and the particle's displacement. Substituting the above equations yields:

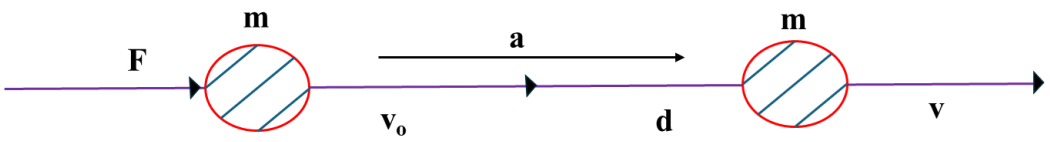
$$w = F \times d$$

$$= ma \times \frac{v_f^2 - v_i^2}{2a}$$

$$= \frac{1}{2} m(v_f^2 - v_i^2)$$

$$= KE_f - KE_i$$

$$\therefore W = \Delta KE \tag{8}$$



Friction

Friction is a force that opposes motion. It converts kinetic energy into thermal energy. The work done by friction is always negative.

$$W = -F \times d$$

Where,

F is the frictional force

d is the distance traveled

4. Application of Work Energy Theorem and Friction

The work-energy theorem, in conjunction with friction, has a wide range of applications in physics and engineering. Here are a few examples:

1. Motion on a Horizontal Surface

Determining stopping distance: Given the initial velocity of an object and the coefficient of friction, we can calculate the distance it takes to come to a complete stop.

Analyzing braking systems: Understanding the work done by friction in brakes helps in designing efficient braking systems.

2. Motion on an Inclined Plane

Calculating final velocity: Given the height of an inclined plane, the coefficient of friction, and the initial velocity, we can determine the final velocity of an object at the bottom of the incline.

Analyzing roller coasters: The work-energy theorem is used to analyze the energy transformations in roller coasters, considering both potential and kinetic energy as well as frictional losses.

3. Work Done by Springs

Calculating spring force: By analyzing the work done by a spring on an object, we can determine the spring constant, and the force exerted by the spring.

Simple harmonic motion: The work-energy theorem is fundamental to understanding the energy transformations in simple harmonic motion systems.

4. Power Calculations

Mechanical power: By combining the work-energy theorem with the concept of power (work done per unit time), we can calculate the power required to perform a specific task.

Additional Considerations

Conservative and Non-Conservative Forces: The work-energy theorem can be applied to both conservative (e.g., gravity, elastic forces) and non-conservative forces (e.g., friction).

Potential Energy: In many cases, potential energy is also involved, and the conservation of mechanical energy can be applied.

Conservation of energy in case of a body falling along a smooth inclined plane:

Let a body of mass m is sliding down a smooth inclined plane of inclination θ with the horizontal PQR. The vertical height from PQR is h (**Fig. 5**).

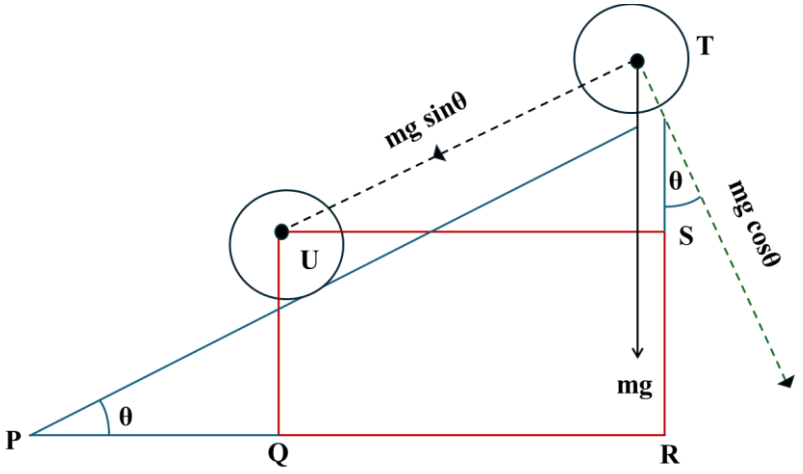


Fig. 5

At T, the kinetic energy of the body is zero (since it is at rest there). So, the potential energy at T,

$$P.E_T = mgh$$

∴ Total energy at T, E = $K.E_T + P.E_T = 0 + mgh = mgh$

Now, suppose the body slides along the inclined plane at U after travelling a distance x. The velocity at U is v. Let US is parallel to PQR and UQ = y.

According to description, the force acting on the body along the inclined plane,

$$F = mg\cos(90° - \theta) = mg\sin\theta$$

Here, acceleration $a = g\sin\theta$ and displacement $s = x$.

∴ Potential energy at U, $P.E_U = mg \times SR = mg(TR - TS) = mg(h - x\sin\theta)$

Kinetic energy at U, $K.E_U = \frac{1}{2}mv^2 = \frac{1}{2}m \times 2gx\sin\theta$ $[since, v^2 = v_0^2 + 2as\ and\ v_0 = 0]$

$$= mgx\sin\theta$$

So, total energy at U $= K.E_U + P.E_U$

$$= mg(h - x\sin\theta) + mgx\sin\theta$$

$$= mgh$$

∴ Total energy at U = total energy at T. So, it is proved that for a body falling freely down the inclined plane the total energy remains constant.